

AEDC-TR-12-F-7



How to Use Calibration Data to Determine Measurement Uncertainty

**James L. Taylor
Aerospace Testing Alliance**

August 2012

Final Report for Period 2 January – 1 July 2012

Statement A: Approved for public release; distribution unlimited.

**ARNOLD ENGINEERING DEVELOPMENT COMPLEX
ARNOLD AIR FORCE BASE, TENNESSEE
AIR FORCE TEST COMMAND
UNITED STATES AIR FORCE**

NOTICES

When US Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Qualified users may obtain copies of this report from the Defense Technical Information Center.

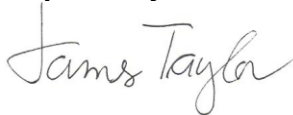
References to named commercial products in this report are not to be considered in any sense as an endorsement of the product by the United States Air Force or the Government.

DESTRUCTION NOTICE

For unclassified, limited documents, destroy by any method that will prevent disclosure of contents or reconstruction of the document.

APPROVAL STATEMENT

Prepared by:



JAMES L TAYLOR
Aerospace Testing Alliance

Reviewed by:



SIMON CHOI, Control Engineering PE
Test Project Manager
TTSI



DEBORAH BAYER
Deputy Director, IT&S Department
Aerospace Testing Alliance

Approved by:



DR. WILLIAM BARNES
Deputy Director
TSDI

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
<p>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.</p> <p>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS</p>					
1. REPORT DATE (DD-MM-YYYY) xx-08-2012		2. REPORT TYPE Final Report		3. DATES COVERED (From – To) 2 Jan – 1 Jul 2012	
4. TITLE AND SUBTITLE How to Use Calibration Errors to Determine Measurement Uncertainty				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Taylor, James L. Aerospace Testing Alliance				5d. PROJECT NUMBER 11748	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Arnold Engineering Development Center Air Force Materiel Command Arnold Air Force Base, TN 37389-6000				8. PERFORMING ORGANIZATION REPORT NO. AEDC-TR-12-F-7	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) TSDI Building 740, 4 th Street, Room 118 Arnold AFB, TN 37389-6000				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.					
13. SUPPLEMENTARY NOTES Available in the Defense Technical Information Center (DTIC).					
14. ABSTRACT This report illustrates two methodologies to establish measurement uncertainty for a family of digital pressure transducers using calibration data. The lumped method assumes calibration data at each input level are statistically independent and lumps all errors together into a single population without regard to pressure level. Descriptive statistics are used to define population parameters that become the basis for measurement uncertainty. The grouped method groups errors according to input level and establishes separate values of measurement uncertainty corresponding to each input level. Both methods produce comparable results and both offer advantages. The lumped method has the advantage of establishing a single measurement uncertainty for measurements made at any input level. The disadvantage is that measurements made at less than midscale are penalized with a higher measurement uncertainty. The grouped method's advantage overcomes the disadvantages of the lumped method by presenting measurement uncertainty as a percent of reading format. The disadvantage is additional complexity in computing measurement uncertainty as a function of input pressure level.					
15. Subject Terms Measurement uncertainty, calibration errors, calibration intervals, out-of-tolerances					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19A. NAME OF RESPONSIBLE PERSON
A. REPORT	B. ABSTRACT	C. THIS PAGE			Simon Choi
Unclassified	Unclassified	Unclassified	Same as Report	24	19B. TELEPHONE NUMBER (Include area code) 931-454-3876

This page is intentionally left blank.

CONTENTS

	<u>Page</u>
1.0 INTRODUCTION	3
1.1 Background	3
1.2 Purpose	3
2.0 Statement of the Problem and Method of Analysis	3
2.1 Problem Statement	3
2.2 Background	4
2.3 Method of Analysis	4
3.0 Results	5
3.1 As-Returned Calibration Errors	5
3.1.1 Individual Transducer Single Calibration	6
3.1.2 Individual Transducer Multiple Calibrations	7
3.1.3 Family of Transducers with Multiple Calibrations	9
3.1.3.1 Lumping All Errors Together Approach	9
3.1.3.2 Analyzing Errors at Each Pressure Approach	10
3.2 As-Received Calibrations.....	11
3.2.1 Establishing Uncertainty by Lumping As-Received Calibration Errors ...	12
3.2.2 Establishing Uncertainty by Grouping As-Received Calibration Errors According to Input Pressure Level	15
3.2.3 Comparing Different Methods for Establishing Measurement Uncertainty.....	17
3.3 Calibration Interval	19
3.4 Measurement Uncertainty	20
3.4.1 Uncertainty Using Pooled As-Received Calibration Data.....	20
3.4.2 Uncertainty Using Percent of Reading Method	21
3.4.3 Additional Considerations	21
4.0 Summary.....	21
REFERENCES	23

FIGURES

Figure

1. Illustration of Calibration Errors in %FS for a Single Calibration7
2. As-Returned Calibration Errors in %FS, Single Transducer, Multiple Calibrations8

3. Standard Uncertainty in %FS at Each Pressure Level	8
4. Error Distribution for Single Transducer	9
5. Digital Pressure Transducer Family of As-Returned Calibration Errors in %FS	9
6. Distribution of As-Returned Errors for Family of Digital Pressure Transducers	10
7. Standard Uncertainty in %FS as a Function of Pressure Level in psf	11
8. Comparison of As-Returned to As-Found Uncertainties in %FS	13
9. Correlation of As-Received and As-Returned Calibration Errors.....	14
10. Distribution of As-Received Errors for Digital Pressure Transducer Family.....	14
11. Comparing As-Received and As-Returned Error Distributions	15
12. Distribution of Normalized As-Received Errors at Different Pressure Levels in psf.....	16
13. Stacking of Calibration Errors at Each Pressure Level to Establish Overall Frequency Distribution	17
14. Comparison of Frequency Distributions Between the Lumped and Pooled Approach	18
15. Expressing Measurement Uncertainty as a Percent of Reading	19
16. Individual Transducer's Number of Out-of-Tolerances.....	20

TABLES

1. Typical As-Returned Calibration Record.....	6
2. Multiple Calibrations for Individual Transducer	7
3. Multiple Calibrations for Individual Transducer	11
4. Measurement Uncertainty at Each Pressure Level	16
5. Comparison of Methods of Establishing Measurement Uncertainty	18
6. Relationship Between Number of OOT Points and Number of OOT Calibrations	20
7. Computed Uncertainty Using Percent of Reading Method	21
NOMENCLATURE.....	24

1.0 INTRODUCTION

1.1 BACKGROUND

Manufacturers use specifications or data sheets to describe the performance characteristics of their products. For a family of pressure transducers, there can be performance variations from transducer to transducer. This is the result of differences in material and variations in the manufacturing process. Because of these variations in a product family, there will be some performers whose performance is much better than the specification, some which are average performers, and some that are performance laggards.

Performance is quantified through a calibration process where a known input is applied using a National Institute of Standards and Technology (NIST) traceable standard. For an instrument such as a pressure transducer, the difference between the measured pressure and the applied pressure is defined as error (Ref. 1). This calibration process is repeated by applying inputs that range from minimum to maximum or full scale. These calibration results can be used to statistically quantify the performance of either a single instrument or of a family of instruments using measurement uncertainty concepts (Refs. 2-4).

There are two classifications of calibration errors -- as-returned and as-received. The terminology used in this report is from a calibration laboratory's perspective. The as-returned errors document the performance at the time the calibration was performed and the item was returned to the user. The as-received errors capture the performance of the instrument after its usage cycle when the item is received back by the calibration laboratory for routine servicing. The difference between the as-returned and the as-received uncertainty is a measure of the growth in uncertainty over the time interval.

The collection of as-received calibration errors for either a single item or a family of like items provides a basis for developing a measurement uncertainty analysis. The as-returned calibration errors may be of interest in establishing baseline performance but are not generally used in defining measurement uncertainty.

1.2 PURPOSE

The purpose of this document is to provide an example for developing measurement uncertainty statements using as-received calibration data. The example is based on a family of digital pressure transducers that are used in AEDC's wind tunnels.

2.0 STATEMENT OF THE PROBLEM AND METHOD OF ANALYSIS

2.1 PROBLEM STATEMENT

Each static or steady-state calibration produces measurement errors corresponding to each discrete calibration point. These individual errors can be expressed in engineering units such as volts, pounds pressure, etc., or they can be expressed in terms of percent of reading or percent of full scale (FS).

The problem addressed in this report involves using as-received calibration data to quantify measurement uncertainty when the calibration data may not be statistically independent. For example, the as-received calibration data from a single calibration may not be statistically independent since gain errors or offset errors affect multiple input levels on the same

calibration. However, calibration data from different calibrations can be considered statistically independent at each input level. There are two different methods that can be used to establish measurement uncertainty for a family of digital pressure transducers with the data, explained as follows:

- All errors from all calibrations and from all input levels can be lumped or grouped together as a single population of errors. This methodology is attractive since it yields a single value of measurement uncertainty. However, this may overestimate the uncertainty at inputs less than midscale and underestimate the uncertainty at inputs greater than midscale.
- The errors at each input level for all calibrations can be grouped together and analyzed separately for each input level. Since the errors at each input level from multiple calibrations are considered independent, this approach yields a value of uncertainty for each different input level. This methodology is attractive since it enables the uncertainty to be presented as a function of the different input levels (e.g., as a percent of reading).

Regardless of which method is used to analyze the calibration data, there is an additional consideration in that measurement uncertainty for some product families increases with time.

For example, a family of items that has been calibrated at 12-month intervals will typically exhibit a larger measurement uncertainty than it would be if the calibration interval had been 9 months. Similarly, the measurement uncertainty at 9 months will be greater than it would be if the interval had been 6 months. The Air Force uses the calibration maintenance data from across the Air Force inventory to actively manage the calibration interval for each product. The stated goal is to achieve an 85% end-of-period reliability (Ref 5). As a result of growth in measurement uncertainty over time, the reported measurement uncertainty must reference the calibration interval. If the interval is changed, the measurement uncertainty must be re-evaluated and changed accordingly.

2.2 BACKGROUND

This study is confined to a family of digital pressure transducers that are rated at 4,000 psf absolute. This group of transducers was chosen because they are principally used to define the operating characteristics for AEDC's wind tunnels. Because of their critical application, their measurement uncertainty is of interest.

There are 18 transducers in the family that were included as part of this analysis. Each transducer had multiple calibrations performed over a 3-year period.

This family is specified to have an accuracy of $\pm 0.010\%$ FS with a manufacturer's recommended calibration interval of 6 months. The manufacturer clarified this specification by stating that the accuracy specification represented a normally distributed variable with $\pm 0.010\%$ FS representing 95% probability.

2.3 METHOD OF ANALYSIS

The process is listed below:

1. Identify family members by property number or other unique identification.

2. Locate PMEL calibration data on the PMEL website.
3. Use 1 to 4 cycles of the most recent as-received calibration data.
4. Create an Excel file for the family with a separate worksheet for each member and a Summary worksheet to collect family results.
5. Copy as-received calibration data and paste values into the worksheet. The as-returned calibration data can also be copied and pasted into the same worksheet if the goal is to understand the growth in measurement uncertainty.
6. Copy and paste as-received errors into the Summary worksheet. Pool all as-received calibration errors and use descriptive statistics to determine average and standard deviation.
7. Compute standard uncertainty using the family's average and standard deviation.
8. An alternate method to pooling all errors is to group all errors by each specific input level. If five input levels are tested for each transducer (e.g., 20% FS, 40% FS, 60% FS, 80% FS, 100% FS), there would be five separate values of uncertainty determined. These could be presented as a table of uncertainty values enabling the user to select the value of uncertainty corresponding to their measurement. An alternate presentation would be to use the five values of uncertainty to determine a straight line.
9. Identify the standard uncertainty of all PMEL calibration standards that are used with this family of items. Use a representative standard's measurement uncertainty that will be combined with the standard uncertainty.
10. Identify the incremental change in uncertainty representing variations in environment or other application related factors that will be added to the above to establish total measurement uncertainty.
11. Establish measurement uncertainty by combining the calibration statistics with both the Calibration Standard measurement uncertainty and an incremental amount representing the environmental effects such as temperature or application operational procedures such as re-zero, re-span, etc.

3.0 RESULTS

3.1 AS-RETURNED CALIBRATION ERRORS

The as-returned calibration errors document the measuring equipment's performance prior to being returned to the user. The calibration is performed by applying different steady-state input pressures using one of PMEL's secondary pressure standards. The standard provides a test uncertainty ratio (TUR) of 2.24:1 (Ref 5). After all calibration adjustments are made to the transducer being calibrated, the as-returned data are collected at a nominal zero and at 5 ascending pressures from 20% to 100% in 20% increments. The data are also collected at these same descending pressure levels and again at nominal zero. At each pressure setting, the error is computed as the difference between the transducer's output and the applied pressure. This process produces 12 error measurements for each as-returned calibration.

3.1.1 Individual Transducer Single Calibration

Table 1 is a partial calibration record for a single pressure transducer. Pressure is applied in ascending pressure steps until full scale is reached. Once full scale is reached, pressure is applied in descending pressure steps until zero is obtained. At each step, data are collected and errors are calculated. The descriptive statistics for this calibration are average-0.000% FS, standard deviation-0.002% FS, and standard uncertainty of 0.002% FS.

Figure 1 illustrates the errors at each input level. The data are shown for both ascending and descending pressure steps. The difference between the ascending and descending pressure steps is a measure of hysteresis.

Table 1. Typical As-Returned Calibration Record

FINAL (AS-RETURNED) CALIBRATION			
Data Points	Standard Pressure psf	Test Instrument Indication psf	Deviation from Standard % F.S.
100.0	100.3896	100.3400	-0.0012
800.0	788.5281	788.4900	-0.0010
1600.0	1593.4679	1593.4600	-0.0002
2400.0	2398.4523	2398.3700	-0.0021
3200.0	3189.0699	3188.9900	-0.0020
4000.0	3993.9189	3993.8600	-0.0015
4000.0	3993.9305	3993.9100	-0.0005
3200.0	3189.1030	3189.1300	0.0007
2400.0	2398.4754	2398.5800	0.0026
1600.0	1593.4737	1593.6300	0.0039
800.0	788.5281	788.6000	0.0018
100.0	100.3637	100.3300	-0.0008

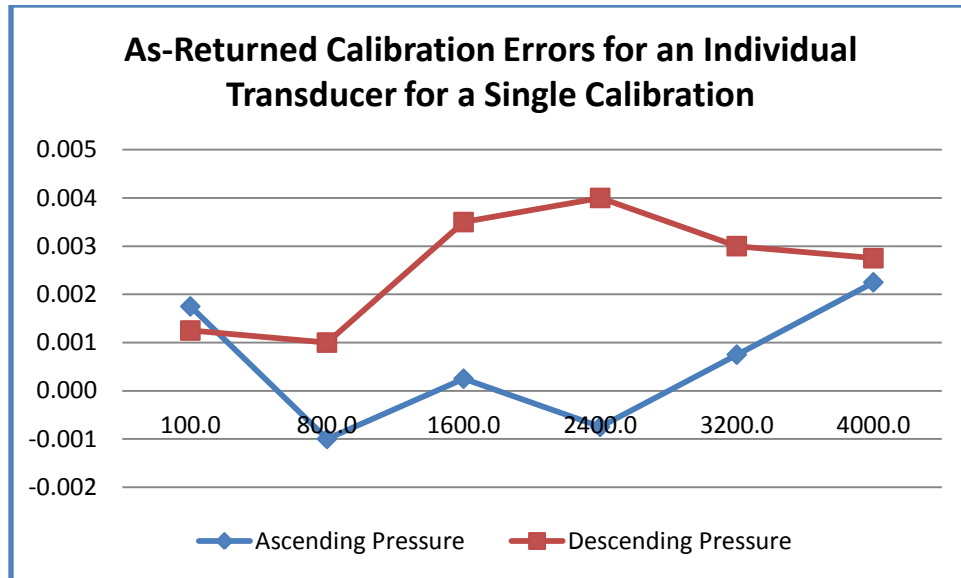


Figure 1. Illustration of Calibration Errors in % FS for a Single Calibration

3.1.2 Individual Transducer Multiple Calibrations

Table 2 summarizes the calibration data from 7 individual calibrations spanning 28 months for 1 transducer. Since these are results obtained after calibration, the error profiles or hysteresis loops are similar. These are shown in Fig. 2. The calibrations are similar with a standard uncertainty for each calibration of 0.002% FS.

Table 2. Multiple Calibrations for Individual Transducer

FINAL (AS-RETURNED) CALIBRATION			
Data Points	Standard Pressure psf	Test Instrument Indication psf	Deviation from Standard % F.S.
100.0	100.3896	100.3400	-0.0012
800.0	788.5281	788.4900	-0.0010
1600.0	1593.4679	1593.4600	-0.0002
2400.0	2398.4523	2398.3700	-0.0021
3200.0	3189.0699	3188.9900	-0.0020
4000.0	3993.9189	3993.8600	-0.0015
4000.0	3993.9305	3993.9100	-0.0005
3200.0	3189.1030	3189.1300	0.0007
2400.0	2398.4754	2398.5800	0.0026
1600.0	1593.4737	1593.6300	0.0039
800.0	788.5281	788.6000	0.0018
100.0	100.3637	100.3300	-0.0008

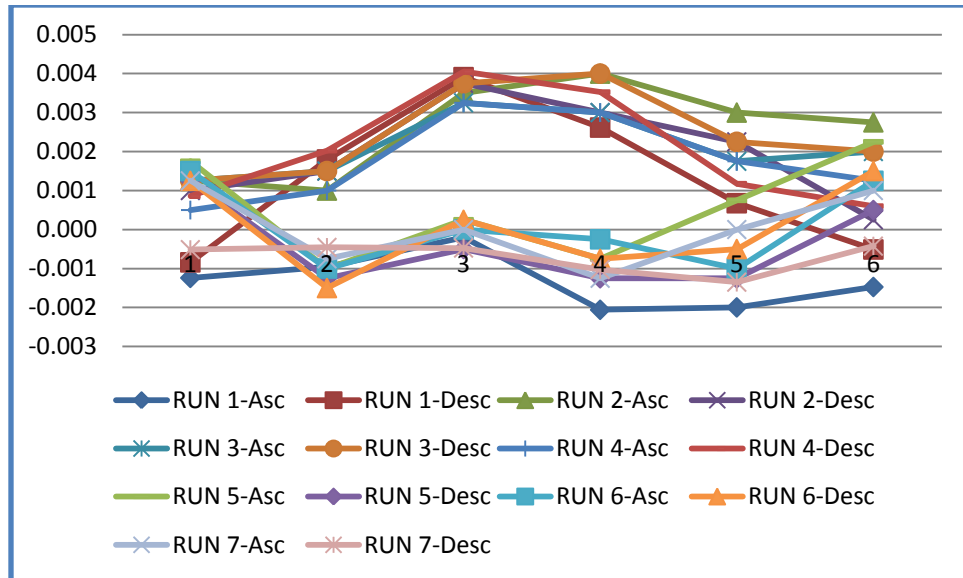


Figure 2. As-Returned Calibration Errors in % FS, Single Transducer, Multiple Calibrations

Figure 3 illustrates the standard uncertainty at each pressure level and was established based on 7 calibrations performed over the 28-month period. Each point represents the standard uncertainty at that pressure level computed using the seven errors from the seven calibrations. The data indicate that the uncertainty is greatest near mid range. This is most likely the result of hysteresis.

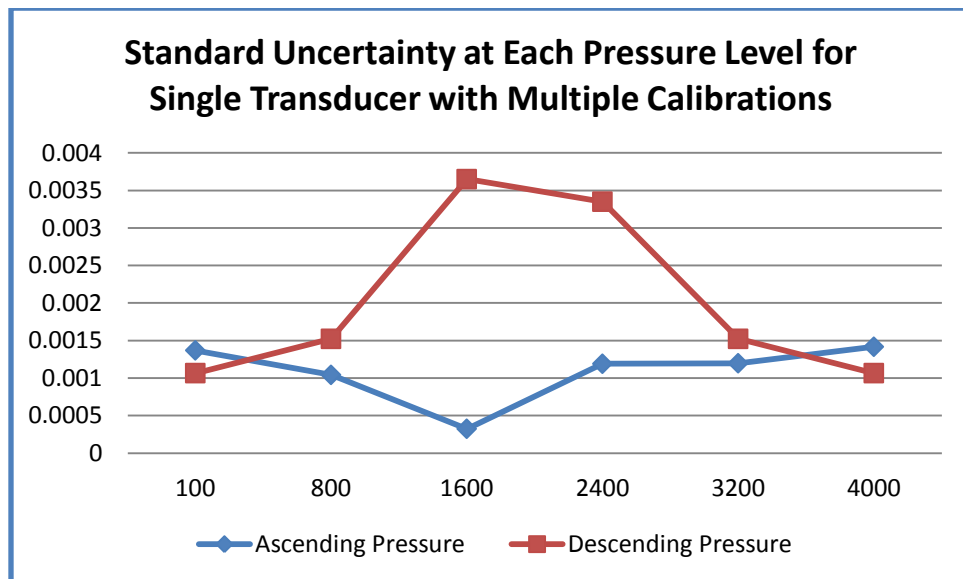


Figure 3. Standard Uncertainty in % FS at Each Pressure Level

Figure 4 illustrates the distribution of errors for this set of seven calibrations. The distribution is not Gaussian; instead, it appears to resemble a rectangular or equal probability distribution. This equal probability distribution is consistent with the calibration objective of minimizing the error across the range of input values. If descriptive statistics are used with Table 2 error data, the results are average 0.0009% FS, standard deviation 0.0016% FS, standard uncertainty 0.0019% FS. There are 84 errors in this sample. If this were a Gaussian or normal distribution,

we would expect approximately 68% of the errors (57 errors) to be within $\pm 1\sigma$ of the average. This data set indicates that approximately 86% of the errors are within $\pm 1\sigma$ of the average, not the expected 68% for a normal distribution. When descriptive statistics are used with distributions such as the error distribution shown in Fig. 4, the results of interpreting probability of errors between any two values can be misleading.

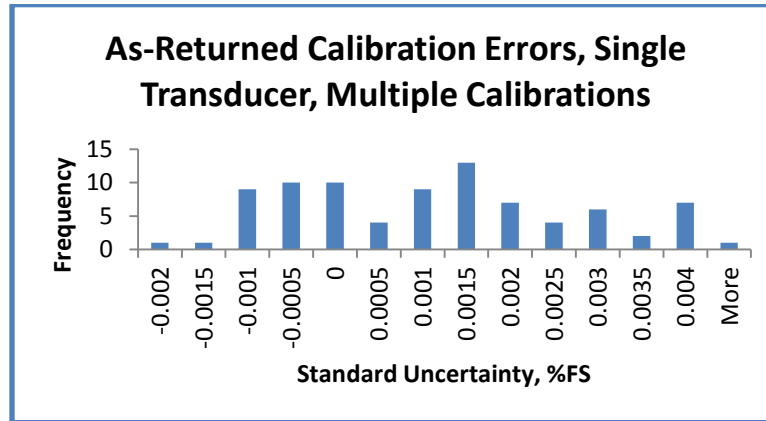


Figure 4. Error Distribution for Single Transducer

3.1.3 Family of Transducers with Multiple Calibrations

3.1.3.1 Lumping All Errors Together Approach

Figure 5 illustrates the expanded uncertainty, U_{95} , for each of the 18 transducers in this family. Each individual transducer's uncertainty was computed using the methodology described previously. As shown, all 18 transducers exhibit some variation but are within the manufacturer's specification of $\pm 0.010\%$ FS. Excluding transducer 5, the remaining 17 members are all less than half of the specification of $\pm 0.01\%$ FS, thus providing margin for uncertainty growth. Anticipating that the uncertainty will grow after calibration, this margin allows each transducer to double in uncertainty and still remain within tolerance.

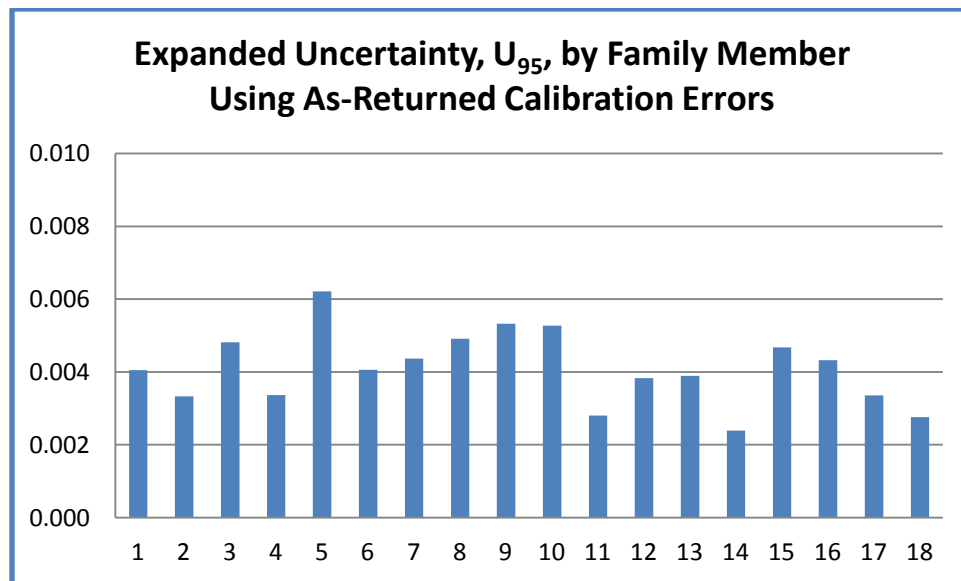


Figure 5. Digital Pressure Transducer Family of As-Returned Calibration Errors in % FS

For this data set consisting of multiple calibrations for each of the 18 members, the standard uncertainty was computed by pooling all errors from each calibration together. This approach assumes that all errors for each transducer are independent. The computed statistics for this family are as follows:

- Average: 0.0007% FS
- Standard Deviation: $\pm 0.0021\%$ FS
- Standard Uncertainty, u_c : 0.0021% FS
- Expanded Uncertainty, U_{95} : $\pm 0.0042\%$ FS
- Number of Errors: 1,530

Figure 6 illustrates the as-returned error histogram for this family of digital pressure transducers. This is based on a population of 1,530 errors. As shown, the result is a distribution that is approximately Gaussian with a slight positive bias.

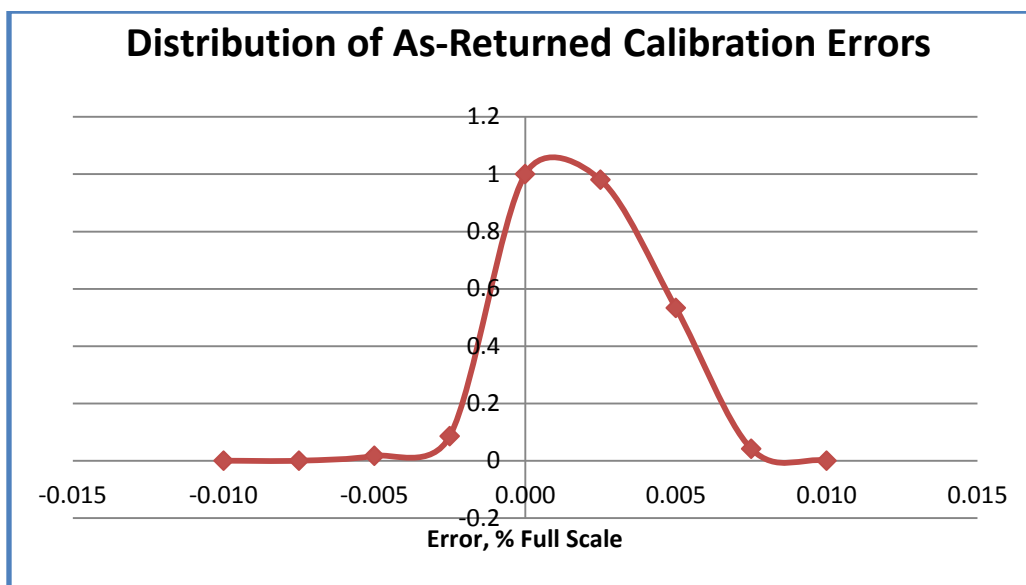


Figure 6. Distribution of As-Returned Errors for Family of Digital Pressure Transducers

3.1.3.2 Analyzing Errors at Each Pressure Level Approach

Figure 7 illustrates the standard uncertainty at each pressure level. These data were determined by combining all errors at each of the five pressure levels.

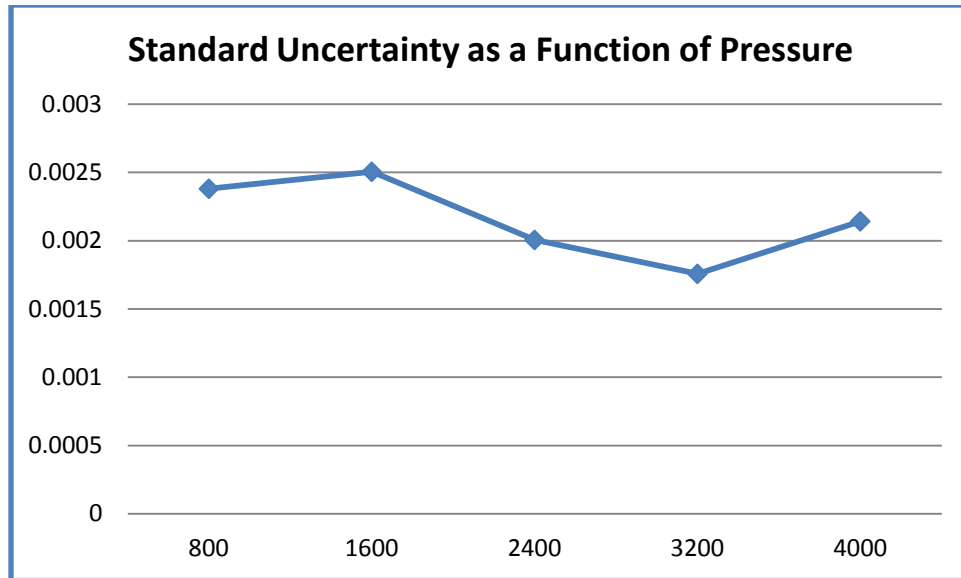


Figure 7. Standard Uncertainty in % FS as a Function of Pressure Level in psf

3.2 AS-RECEIVED CALIBRATIONS

The analysis is based on collecting the calibration errors for each of the 18 transducers in this family for as many as 10 calibrations over a 3-year period. A single transducer in this family can have 50 individual measures of error, 5 errors for each calibration. Table 3 illustrates typical as-received data for a single transducer for six different calibrations. As shown, data are collected at five ascending pressure steps. In general, the errors for each calibration tend to increase as pressure is increased. The calibration dated 20081218 indicates that both an offset and gain error occurred since all five pressure readings for that calibration have increased. These as-received data suggest that the data at each calibration are not independent and should not be lumped together. However, since the pressures at each pressure step can be considered independent, grouping all errors at each pressure level together is appropriate. This would produce a different value of measurement uncertainty for each of the five input levels.

Table 3. Multiple Calibrations for Individual Transducer

As Received						
Data	Calibration Date, Year/Month/Day					
Points	20080205	20080529	20080918	20081218	20090309	20090610
800	-0.003	-0.003	0.000	-0.006	-0.002	0.001
1600	0.000	-0.003	0.003	-0.007	-0.001	0.004
2400	-0.001	-0.005	0.006	-0.010	-0.001	0.005
3200	0.001	-0.006	0.007	-0.013	-0.001	0.007
4000	0.004	-0.003	0.012	-0.014	0.002	0.011

The approach described in Section 3.1 for as-returned errors where all errors are lumped together assumes that the errors are statistically independent. Since adjustments are made as part of the calibration, the assumption that errors caused by offset or gain changes do not exist is appropriate. However, this would not be a reasonable assumption when analyzing as-

received errors since multiple pressure settings can be affected by a single error such as offset or gain. This is illustrated with the data in Table 3.

The measurement uncertainty of interest is computed at the end of the usage period using as-received calibration data and not the uncertainty computed at the beginning of a usage period using as-retained calibration data. Since measurement uncertainty typically grows with time, the standard uncertainty computed using as-received calibration data will likely increase with time. If the calibration interval is changed, then the measurement uncertainty will likely change as well.

Manufacturers often provide a recommended calibration interval such as 6 months or a year. For AEDC, the manufacturer's recommended calibration interval represents the initial interval. The calibration interval is actively managed by AFMETCAL to achieve a desired 85% end-of-period reliability. At the end of the interval when the item is recalled for periodic calibration, the as-received errors are determined prior to calibration. These individual errors are combined to form the as-received error distribution.

The as-received errors for a product family are of interest because they can be used to establish the measurement uncertainty for the family. The data from the different calibrations can be grouped together as a single population, or the data can be grouped according to input levels. Both methods are discussed below.

3.2.1 Establishing Uncertainty by Lumping As Received Calibration Errors

The collection of as-received calibration errors for a product can be combined into a single population and descriptive statistics established. When statistics based on as-received calibration data are used as the basis for published measurement uncertainty, a clarification should be provided that stipulates that the as-received data are based on a specific calibration interval. The reason is that the measurement uncertainty typically grows with time and is at its minimum value when calibrated.

In all likelihood, there will be a difference between the as-retained and the as-received error distributions. This difference illustrates uncertainty growth. The growth is the result of individual changes that occur with each member of the family over the calibration interval time. Since each individual item can have different growth rates, it is probable that there will be some performance laggards that are out of tolerance. These can significantly affect the as-received errors. If the as-received errors exceed the manufacturer's specification, it may be necessary to reduce the calibration interval to maintain the desired end-of-period reliability. In contrast, it may be possible to increase the calibration interval if the population has only a few performance laggards.

For the family of digital pressure transducers that is the subject of this report, the as-received errors are determined at five different pressure levels ranging from 20% to 100% FS. These data are captured before any adjustments are made other than the zero settings. The zeroes are set prior to capturing the as-found data since this is the operational process in use.

Figure 8 illustrates both the as-retained and as-received calibration errors for each of the 18 transducers in this family. For each transducer, the difference between the as-retained and as-received U_{95} expanded uncertainties illustrates the growth attributable to time and use. As shown, the growth differs for the different transducers. For consistency, only the last 9

calibrations from each of the 18 transducers were used. The average interval between calibrations was between 3 and 4 months.

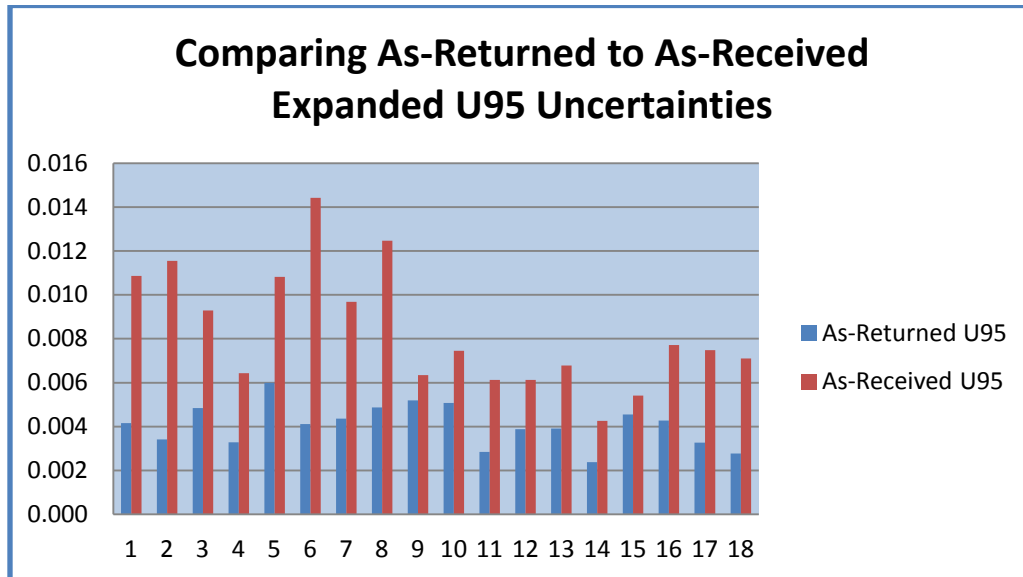


Figure 8. Comparison of As-Returned to As-Found Uncertainties in % FS

The set of as-received data consisting of 5 pressures for each of 18 transducers for up to 9 calibrations for each transducer is used to define the performance. The descriptive statistics for this set are as follows:

- Average: -0.0003% FS
- Standard Deviation: 0.0043% FS
- Standard Uncertainty: 0.0043% FS
- Expanded Uncertainty, U_{95} : $\pm 0.0086\%$ FS
- Number of Samples: 805

Figure 9 illustrates the correlation between the as-returned and as-received calibration errors for this family of digital pressure transducers. The correlation is 0.38. The data indicate that the value of as-returned standard uncertainty is not a good predictor of the growth in as-received uncertainty.

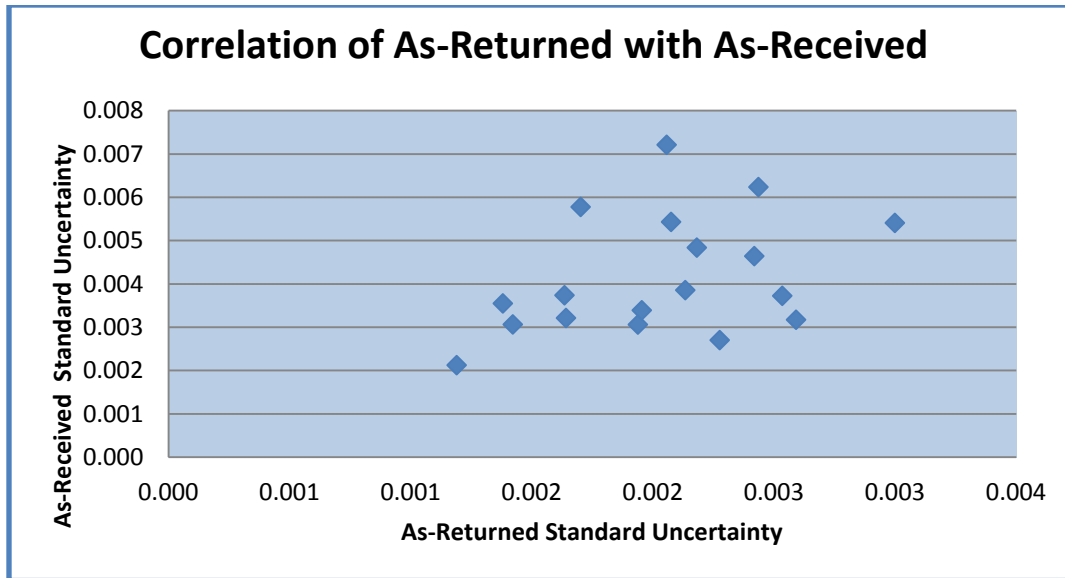


Figure 9. Correlation of As-Received and As-Returned Calibration Errors

Figure 10 illustrates the histogram of the as-received errors. This is a bi-modal distribution with a primary peak at zero and a secondary peak at 0.005% FS. This set of as-received data can be used as the basis for establishing measurement uncertainty. The uncertainty of the calibrations standards, an allowance for the effects of the environment, and an allowance for application-related factors must be added to the above calculation of expanded uncertainty to quantify the measurement's uncertainty.

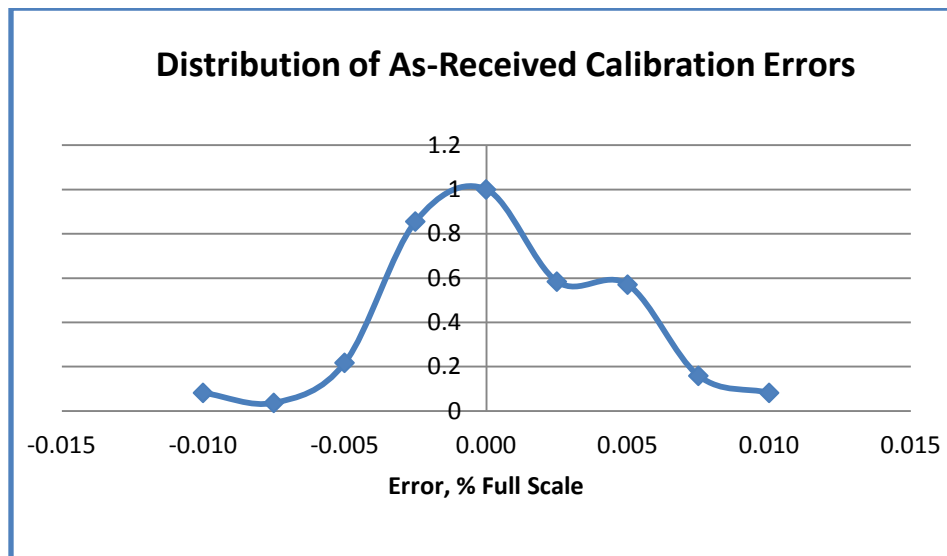


Figure 10. Distribution of As-Received Errors for Digital Pressure Transducer Family

Figure 11 compares the as-returned to as-received frequency distributions. The as-received data illustrate increased variability and not linear drift. The increase in U_{95} from the as-returned to as-received is a factor of approximately 2. This indicates that if the calibration interval were extended, the variability in the as-received data would likely increase. This would result in an increased number of errors that were outside the tolerance specification of $\pm 0.010\%$ FS, leading to a reduction in the calibration interval.

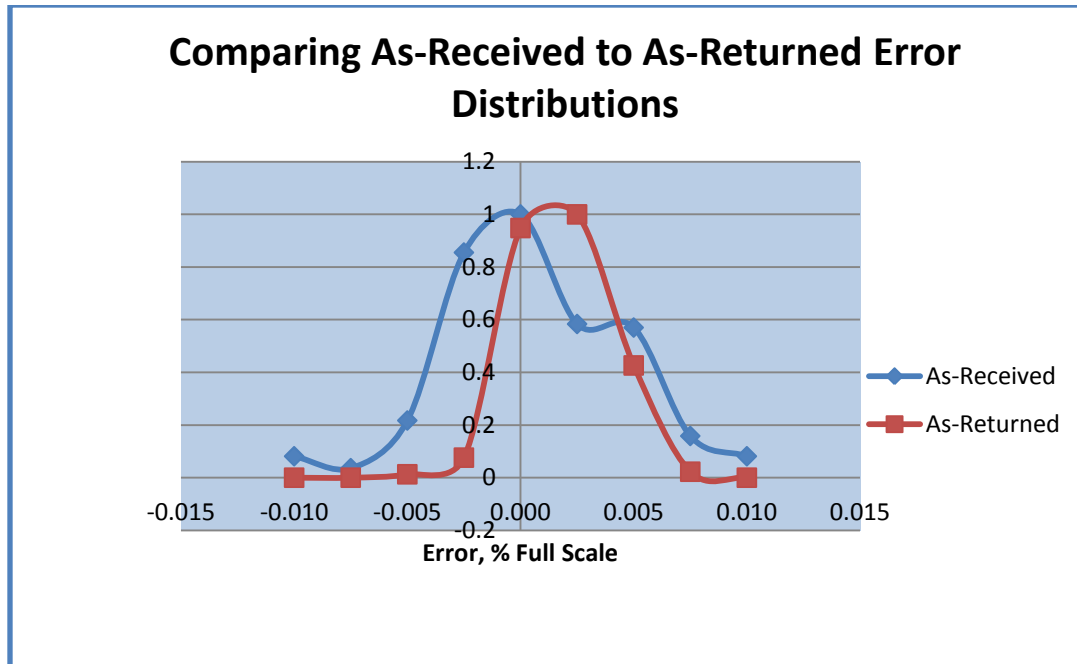


Figure 11. Comparing As-Received and As-Returned Error Distributions

3.2.2 Establishing Uncertainty by Grouping As-Received Calibration Errors According to Input Pressure Level

An alternative to combining all errors together without regard to input level is to analyze the performance at each specific input level. For this digital pressure transducer family, the as-received data are collected at five pressure levels. There are 160 measures of error at each of the 5 pressure levels.

Figure 12 illustrates the distribution of measurement errors at each of the five pressure levels. As pressure increases, the variability in errors increases.

Table 4 is a compilation of the statistics at each pressure level. The data indicate that the standard uncertainty increases with pressure ranging from 0.0041% FS at 800 psf to 0.0054% FS at 4,000 psf. The column headed "Averages" provides the average for both the Average and Standard Deviation rows. Standard uncertainty and expanded uncertainty are based on the averages.

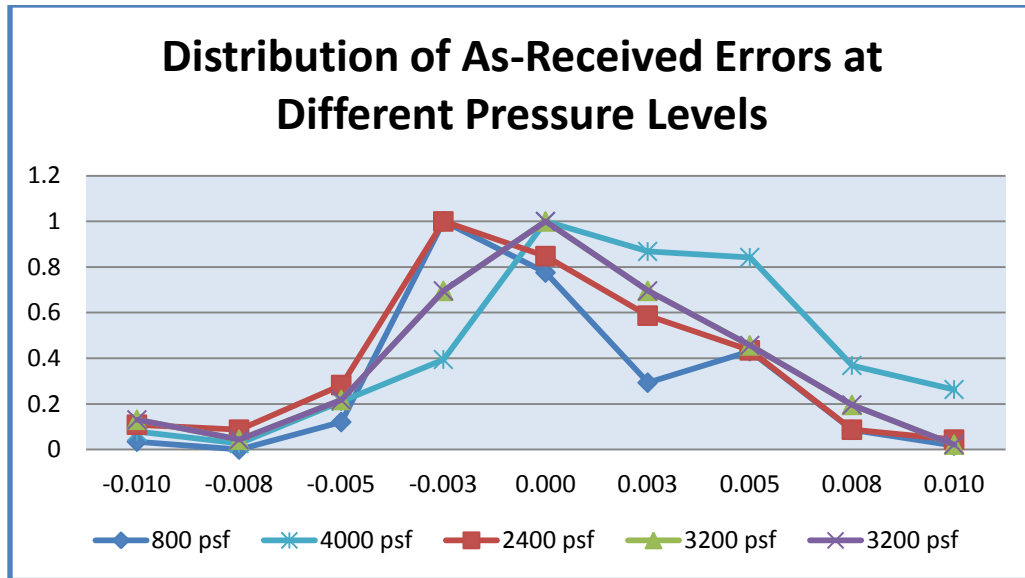


Figure 12. Distribution of Normalized As-Received Errors at Different Pressure Levels in psf

Table 4. Measurement Uncertainty at Each Pressure Level

Measurement Uncertainty at Each Input Level						
	800 psf	1600 psf	2400 psf	3200 psf	4000 psf	Averages
Average,%FS	-0.001	0.000	-0.001	-0.001	0.002	-0.0003
Standard Deviation, %FS	0.004	0.004	0.004	0.004	0.005	0.0042
uc, %FS	0.004	0.004	0.004	0.004	0.005	0.0042
U95	0.007	0.008	0.008	0.009	0.010	0.0084
Count	160	160	160	160	160	

Figure 13 illustrates the combined frequency distribution when the individual frequency distributions for each of the five pressure levels are combined to form a single distribution.

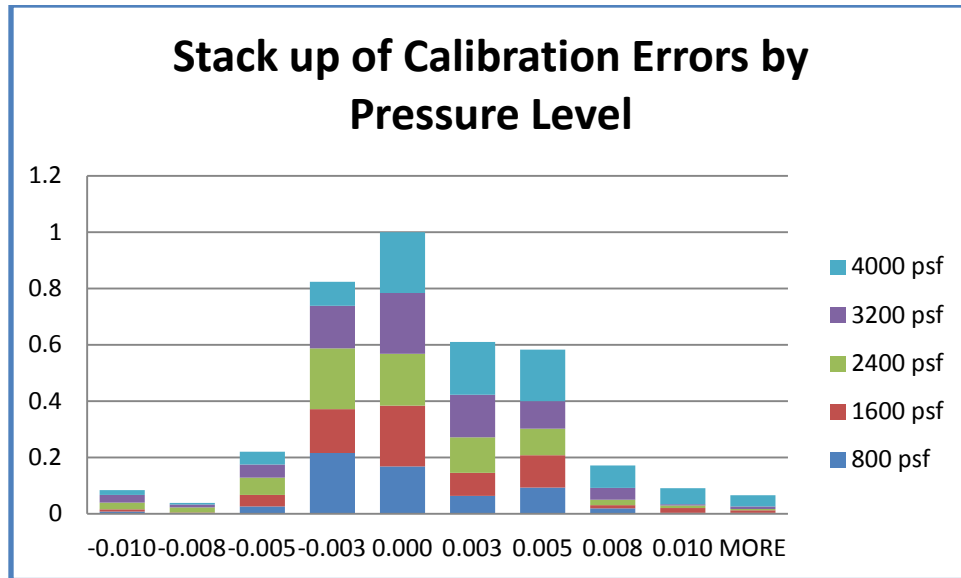


Figure 13. Stacking of Calibration Errors at Each Pressure Level to Establish Overall Frequency Distribution

3.2.3 Comparing Different Methods for Establishing Measurement Uncertainty

The two methods discussed in this report include the (1) lumped method which combines all errors without regard to input level and (2) the grouped method which establishes uncertainty at each input level and then averages these to establish an overall measurement uncertainty. Table 5 compares the two methods and indicates that the differences are small.

Figure 14 is an overlay of the lumped and grouped distributions. The lumped distribution considers all errors to be from the same distribution without regard to pressure level. The grouped distribution is the sum of the individual distributions for each of the five pressure levels. As shown in this graphic, the two distributions appear identical and should be since both utilize the same data.

The grouped approach, which establishes uncertainty at each input level, opens up the opportunity to present the measurement uncertainty in a traditional percent-of-reading format. Figure 15 illustrates this using the expanded U_{95} measurement uncertainty from Table 4. Using the two end points (800 psf and 4,000 psf), a straight line is constructed of the form $Y = mX + b$, where Y is expanded measurement uncertainty in % FS, m is the slope with value $9.13E-07$, and b is the intercept with value 0.00675.

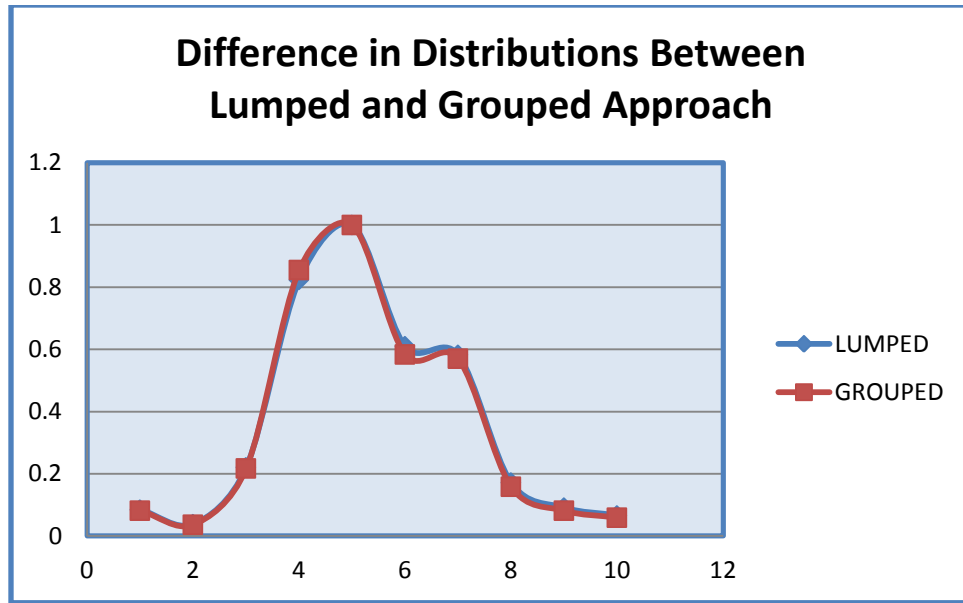


Figure 14. Comparison of Frequency Distributions Between the Lumped and Pooled Approach

Table 5. Comparison of Methods of Establishing Measurement Uncertainty

	Lumped Method	Grouped Method	Difference
Average,%FS	-0.0003	-0.0003	0.0000
Standard Deviation, %FS	0.0043	0.0042	0.0001
u_c , %FS	0.0043	0.0042	0.0001
U_{95}	0.0086	0.0084	0.0002

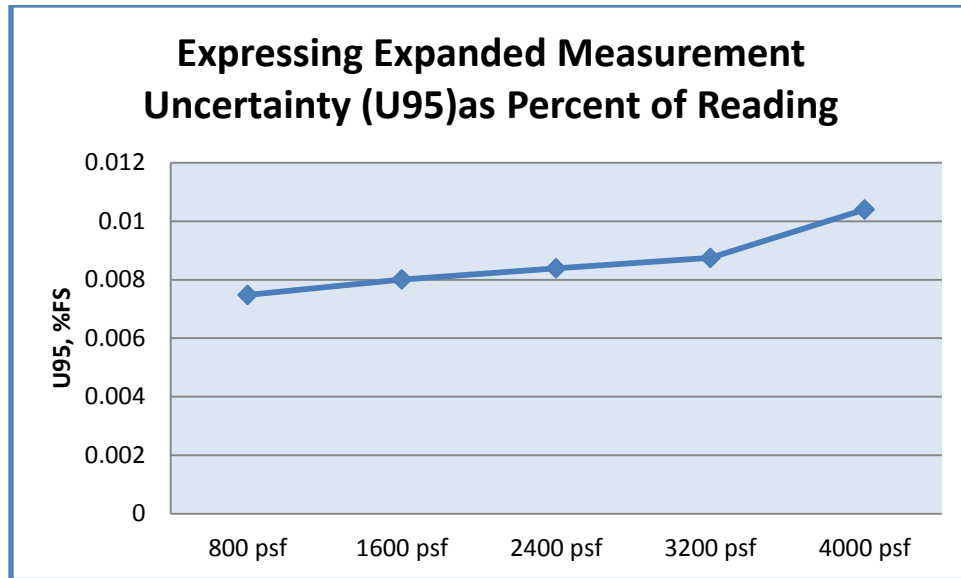


Figure 15. Expressing Measurement Uncertainty as a Percent of Reading

3.3 CALIBRATION INTERVAL

The manufacturer's recommended calibration interval for this product is 6 months. From the time the product was initially placed in service, the calibration interval has been purposely reduced to maintain an end-of-period reliability of 85%. The average calibration interval for the data presented in this document is 3.5 months. A review of the performance data indicates that there are several transducers that are frequently out of tolerance, and these are driving the interval. Removal of these transducers will enable the interval to increase.

Figure 16 illustrates the number of out-of-tolerances for each of the 18 transducers. As shown, there are a few that account for most of the out-of-tolerances. Table 6 describes the numbers of out-of-tolerance points and the number of out-of-tolerance calibrations for the four transducers. Since Transducer 2 has 5 out-of-tolerances resulting in only one bad calibration, this can be considered an isolated instance where the transducer malfunctioned during the usage process.

Transducer 1, 6, and 8 have several bad calibrations and should be removed from service to avoid impacting the calibration interval. If these four outliers (Transducers 1, 2, 6, and 8) were removed from service, the expanded uncertainty would decrease from $U_{95} = \pm 0.0086\%$ FS to $U_{95} = \pm 0.007\%$ FS. While this is not a significant reduction in the expanded uncertainty, it does remove those transducers that are causing the interval to be reduced because of their excessive out-of-tolerances. For example, 9 of the 12 calibrations with out-of-tolerances are attributable to these 4 transducers. If they were not in service, there would only have been 3 calibrations out of 125 calibrations that had one or more out of tolerances.

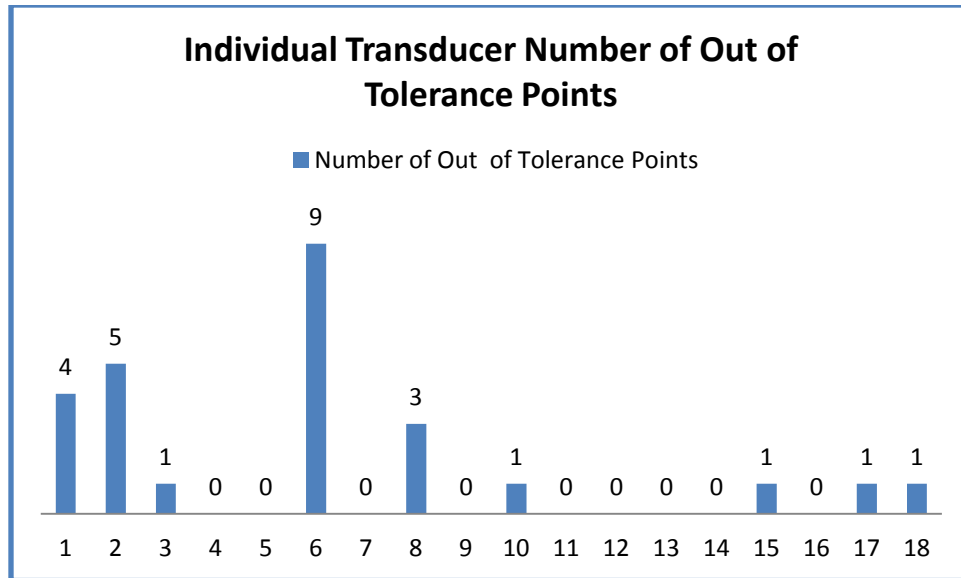


Figure 16. Individual Transducer's Number of Out of Tolerances

Table 6. Relationship Between Number of OOT Points and Number of OOT Calibrations

TRANSDUCER NUMBER	NUMBER OF OOT POINTS	NUMBER OF OOT CALIBRATIONS
1	4	3
2	5	1
6	9	3
8	3	2

3.4 MEASUREMENT UNCERTAINTY

For organizations such as AEDC that adjust calibration intervals based on maintenance data, there is a risk that the published uncertainty based on a specific calibration interval will be incorrect. For example, should the interval be decreased as a result of poor performance, the actual uncertainty will be less than the published value. However, should the interval be increased because of outstanding performance, the actual uncertainty will exceed the published uncertainty.

3.4.1 Uncertainty Using Pooled As-Received Calibration Data

The measurement uncertainty for this family of digital pressure transducers can be established using the as-received statistics combined with the standard uncertainty for the PMEL calibration standards used and an allowance for any environmental impact that may result from using the transducers at temperatures that exceed the manufacturer's specification. Additionally, since the user periodically adjusts the transducer zero (barometric pressure) as part of operational procedures, the uncertainty of the barometer that is being used must be included.

The different components of measurement uncertainty are listed below in terms of standard uncertainty. Both the barometric pressure and the PMEL calibration standards have been converted from a 95% expanded uncertainty to standard uncertainty. All are assumed to be independent. The full scale for the barometric pressure transducer is 2,400 psf, and the

standard uncertainty is 0.0125% FS or 0.3 psf. It is assumed that the transducers are re-zeroed frequently to avoid errors in the zero.

- Transducer As-Received Errors (Lumped Method): 0.0043% FS (± 0.17 psf)
- PMEL Calibration Standards: ± 0.05 psf
- Barometric Pressure: ± 0.3 psf

The combined standard uncertainty using RSS is 0.35 psf. For a 95% confidence level, the expanded uncertainty is ± 0.7 psf.

3.4.2 Uncertainty Using Percent of Reading Method

Table 7 lists the uncertainty at each of the five pressure levels and was computed using the straight line presented in Section 3.2.3. For this set of data, there is not a significant difference in uncertainty at each of the pressure levels. The added complexity of using the percent of reading method over the lumped approach may not justify the more complex approach.

Table 7. Computed Uncertainty Using Percent of Reading Method

COMPUTED USING $Y = mX + b$, $m=9.13E-07$, $b=0.00675$					
INPUT PRESSURE, psf	800	1600	2400	3200	4000
uc, psf	0.152	0.164	0.18	0.196	0.2
PMEL Cal Standards, psf	0.05	0.05	0.05	0.05	0.05
Barometric Zero, psf	0.3	0.3	0.3	0.3	0.3
Computed uc in psf	0.34	0.35	0.35	0.36	0.36
Expanded Uncertainty in psf, U95	0.68	0.69	0.71	0.72	0.73

3.4.3 Additional Considerations

There is an additional consideration in establishing measurement uncertainty for this family of digital pressure transducers — the effects of changes in calibration interval on as-received data. Once the four transducers that are causing the majority of out-of-tolerances are removed, the interval will gradually increase over time. It is expected that the interval will increase from the current 3-month interval to as much as 9 months before measurement uncertainty growth causes significant out-of-tolerances. It is expected that the number of out-of-tolerances at a 9-month interval will drive the interval back down to 6 months.

4.0 SUMMARY

The as-received calibration data can be used to establish measurement uncertainty in two different ways. First, the uncertainty analysis can be performed for a family of the same instruments by pooling their calibration errors together. Second, the analysis can be performed

at each input level. If as-received calibration data are obtained in 20% increments over the range, the data at each input are considered independent. This latter method has advantages in that the uncertainty can be presented as a function of input, thus permitting reduced estimates of measurement uncertainty at the lower inputs.

Regardless of which method is used, the uncertainty of PMEL's calibration standards and the application's environment, especially temperature, must be included in the measurement uncertainty. For this family of digital pressure transducers, the operational practice is to periodically set zeroes by comparing the transducer's output at ambient pressure to a barometric standard. As a result, the uncertainty of the barometric pressure standard must be included as part of the total measurement uncertainty.

The expanded 95% measurement uncertainty for this group of 4,000 psfa digital pressure transducers using the lumped approach is ± 0.7 psf or $\pm 0.018\%$ FS. This is based on a 3.5-month interval. If the interval is changed, the element of uncertainty that is based on as-received calibration data must also be changed.

This family of digital pressure transducers is limited to an interval of 3-4 months as a result of their performance. Because these transducers were chosen for commonality throughout the facility, they are used in various applications, not all of which require the high accuracy of $\pm 0.010\%$ FS. Accordingly, there is an opportunity for increasing the calibration interval by separating the facility pressure requirements into high-accuracy and low-accuracy requirements. The specific recommendations are as follows:

- Review the measurement uncertainty requirements for each pressure that is being measured using this family of digital pressure transducers, and separate the requirements into high-accuracy ($\pm 0.010\%$ FS, 95% confidence level) and low-accuracy ($\pm 0.025\%$ FS, 95% confidence level) categories.
- Remove Transducers 1, 2, 6, and 8 from the pool of high-performance transducers. Consider classifying these four transducers with a different model or part number and with a de-rated accuracy of $\pm 0.025\%$ FS, 95% confidence. Use visual marking and/or labeling to mitigate the risk of using these in a high-accuracy requirement.

REFERENCES

1. *International Vocabulary of Metrology — Basic and General Concepts and Associated Terms (VIM)*. Committee for Guides in Metrology JCGM 200:2008, 3rd Edition, 2008.
2. “Assessment of Experimental Uncertainty With Application to Wind Tunnel Testing.” S-071A-1999, American Institute of Astronautics, 1801 Alexander Bell Drive, Suite 500, Reston, VA 20191.
3. Test Uncertainty, ASME PTC 19.1-2005, American Society of Mechanical Engineers, 3 Park Avenue, New York, NY 10016-5990.
4. Taylor, Barry N. and Kuyatt, Chris E. “Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results.” NIST Technical Note 1297, 1994 Edition. United States Department of Commerce Technology Administration, National Institute of Standards and Technology.
5. TO 00-20-14, Technical Manual, Air Force Metrology and Calibration Program, 30 September 2011.

NOMENCLATURE

AEDC	Arnold Engineering Development Complex
AFMETCAL	Air Force Metrology Calibration Program
FS	Full scale
NIST	National Institute of Standards and Technology
OOT	Out-of-tolerance
PMEL	Precision Measurement Equipment Laboratory
psf	Pounds square foot
psfa	Pounds square foot absolute
T.I.	Test Instrument
TUR	Test Uncertainty Ratio
u_c	Standard uncertainty
U_{95}	95% confidence interval for measurement uncertainty
σ	Greek symbol for sigma representing the standard deviation of a population